

Unit 18

Statistical Inference: Tests of Hypothesis

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Learning Objectives



It is expected that after reading Unit 18 you would be able to

- ❖ Draw statistical inferences on the basis of the concept of probability
- ❖ Use the tool of statistical inference to test hypotheses
- ❖ Apply the tool of statistical inference for estimating the unknown parameter of the population under research.

18.1 Introduction

Unit 18 deals with statistical inference, which uses the concepts of probability[®] to explain the element of uncertainty in decision-making. You would find that though it occupies a lower status among statistical tests, you would be able to use chi-square test in a wide variety of researches. If you have a relatively smaller sample, it would be better to use student's test that is a parametric[®] test. You would learn in Unit 18 in detail about both the chi-square and student's tests. For hypothesis testing[®], Unit 18 is going to prove to be most helpful in the mini research project that you have to complete as a part of your assignment of MSO 002.

18.2 Statistical Inference

Statistical Inference uses the concept of probability to deal with uncertainty in decision-making. It refers to the process of selecting and using a sample statistics to draw inferences about a population parameter, based on a sample drawn from the population. Statistical inference takes care of the two classes of problems.

- A. Hypothesis testing: It tests some hypotheses about the parent population based on the sample drawn from the population.
- B. Estimation: It uses the 'statistics' obtained from the sample as an estimate of the unknown 'parameter' of the population based on the sample drawn from the population.

A. Hypothesis testing

It begins with an assumption called a hypothesis that one makes about a population parameter.

Steps in testing a hypothesis

- i) Formulate a hypothesis
- ii) Decide an appropriate significance level
- iii) Select a test criterion
- iv) Carry out calculations
- v) Make Decisions

Let us discuss in brief each of the five steps.

i) Formulate a hypothesis: First of all a hypothesis is set up about a population parameter. Thereafter, sample data is collected, sample statistics calculated and the information is used to assess how far the hypothesised parameter is correct. Examining the difference between the hypothesised value and the actual value of the sample's mean tests the validity of an assumption.

Conventionally, rather than a single hypothesis two are constructed. These hypotheses are constructed in such a way that if one hypothesis is accepted the other is rejected. The two hypotheses are called:

- a. Null hypothesis (designated as H_0)
- b. Alternative hypothesis (designated as H_A)

In the simplest form, a null hypothesis states that there is no true difference between the sample statistics and the population parameter. It asserts that the observed difference is accidental and / or unimportant arising out of the fluctuations in sampling.

A researcher, for instance, who wishes to test whether the annual per capita income in a community is higher than Rs. 10,000/- might formulate the null and alternate hypotheses as under.

Null hypothesis (H_0): $\mu = 10,000$

Alternative hypothesis (H_A): $\mu > 10,000$

In another instance, a researcher might wish to test the mean difference between the annual per capita incomes of two groups. In this case, she might formulate the null and alternate hypotheses as under.

Null hypothesis (H_0): $\mu_1 - \mu_2 = 0$

Alternative hypothesis (H_A): $\mu_1 - \mu_2 \neq 0$

ii) Decide an appropriate significance level: The next step in testing hypotheses is to set up a suitable significance level to test the validity of H_0 as against H_A . The confidence with which a null hypothesis is adopted or rejected depends on the adopted significance level.

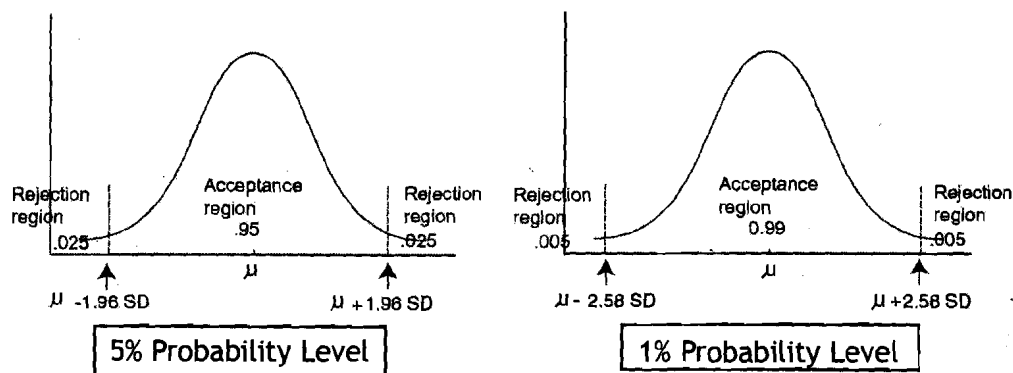


Figure 18.1 Acceptance (or rejection) of Null hypothesis (two tailed) at 5% and 1%, respectively

Conventionally, the significance level is expressed as a percentage, such as 5% or 1%. In the former case, it would mean that there is 5% probability of rejecting a null hypothesis, even if true. This means that there are 5 out of 100 chances that the investigator would reject a true hypothesis (see Figure 18.1).

iii) **Select a test criterion:** The next step in hypothesis testing is to set up a test criterion. An appropriate probability distribution that can be applied is selected for the particular test. Some of the common probability distributions are χ^2 , t and F.

iv) **Carry out calculations:** Computation is carried out of various statistics and their standard errors based on sample.

B. Make Decisions: In this step statistical conclusions and decisions are made to reject or accept the null hypothesis, depending on whether the computed value falls in the region of acceptance or rejection (See Case 1 and Case 2 in 18.3).

Reflection and Action 18.1

Let us say that you are carrying out a research that has both the null and alternative hypotheses. You need now to set up a suitable significance level to test the validity of null hypothesis as against alternative hypothesis. For this task as well subsequent tasks, follow the procedure given in the text. Next, you would need to set up a test criterion. For this purpose select an appropriate probability distribution that can be applied for the particular test. Then carry out computation of various statistics and their standard errors. Now, based on sample statistical conclusions, make decisions to reject or accept the null hypothesis. This would depend on whether the computed value falls in the region of acceptance or rejection. Work out the steps in concrete terms of your own research project and incorporate them in your research work report.

18.3 Cases

Case 1: If the hypothesis is being tested at 5% level and the observed result has a probability of less than 5%, then the difference between the sample statistics and the population parameter is significant and cannot

be explained by chance alone. Thus the null hypothesis (or H_0) is rejected, and in turn, the alternative hypothesis (H_A) is accepted.

Case 2: If the hypothesis is being tested at 5% level and the observed result has a probability of more than 5%, then the difference between the sample statistics and the population parameter is not significant and can be explained by chance variation. Thus the null hypothesis (or H_0) is accepted, and in turn the alternative hypothesis (H_A) is rejected.

In hypothesis testing it is important to understand the following:

- i) One tailed and two tailed test of hypothesis; and
- ii) Type I and Type II errors

i) One-tailed and two-tailed test of hypothesis

Depending on the research problem, the null and alternate hypotheses are defined in such a way that the test is known as one-tailed or two-tailed. A two-tailed test of hypothesis will reject the null hypothesis if the sample statistic is significantly higher or lower than the population parameter. Thus, in a two-tailed test of hypothesis the rejection region is located on both the tails and the size of the rejection region is .025, whereas the central acceptance region is .95 (Fig.18 2). If the sample mean falls within $\mu \pm 1.96$ SD (i.e. in the acceptance region), the hypothesis is accepted. If on the other hand, it falls beyond $\mu \pm 1.96$ SD, then the hypothesis is rejected, as it will fall in the rejection region.

Let us take an example of the two-tailed hypothesis. Suppose a researcher is interested in knowing whether there is gender difference in IQ. You can formulate the following hypotheses.

IQ of Females = IQ of Males (Null hypothesis)

IQ of Females \neq IQ of Males (Alternative hypothesis) or in other words, IQ of females may be lower or higher than that of males.

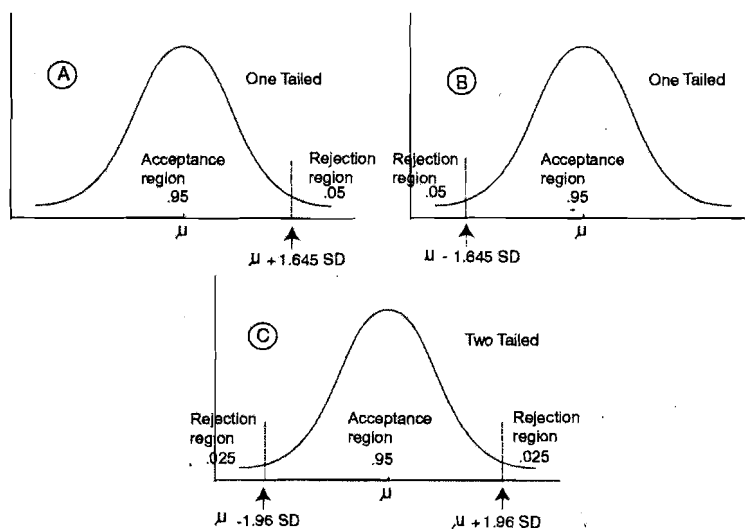


Figure 18.2 One-Tailed and Two-tailed Test of Hypothesis. (A) and (B) are One-Tailed, whereas (C) is Two-tailed.

In contrast to the two-tailed hypothesis, in one-tailed hypothesis the rejection region will be located only on one tail (see Figure 18.2). In this case, the size of the rejection region will be .05, if one is testing the hypothesis at 5% probability level. If the sample mean falls above $\mu + 1.645$ SD (Case A: Fig. 2) or below $\mu - 1.645$ SD (see Case B of Figure 18.2), then the hypothesis is rejected, as it will fall in the rejection region.

Let us take an example of the one-tailed hypothesis. Suppose a researcher is interested in knowing whether the IQ of females is higher than that of males. In this case, you can formulate the following hypotheses.

IQ of Females > IQ of Males (Null hypothesis)

IQ of Females = IQ of Males (Alternative hypothesis)

ii) Type I and Type II errors

i) A researcher's decision is correct when a true hypothesis is accepted and the false hypothesis is rejected. One-tailed and two-tailed test of hypothesis; and

ii) Type I and Type II errors

	Accept H_0	Reject H_0
H_0 is True	Correct Decision	Type I Error
H_0 is False	Type II Error	Correct Decision

Figure 18.3 Type I and Type II Errors in Testing a Hypothesis.

The Type I error is designated as α (alpha), whereas Type II error is designated as β (beta). It is important to note that both types of errors cannot be reduced simultaneously, as reduction in one leads to increase in the other if the sample size remains unchanged. Thus if Type I error decreases, Type II error will increase. In most of the statistical tests, the level of significance is fixed at 5% probability level (= 0.05). This means that the probability of accepting a true hypothesis is 95%. Sometimes, the level of significance is fixed at 1% probability level (=0.01). In that case, the probability of accepting a true hypothesis is 99%. In this case, accepting a false hypothesis will also increase.

Reflection and Action 18.2

Carry on with hypothesis testing with the same example as you had taken in Reflection and Action 18.1. You need now to carry out both one-tailed and two-tailed tests of your hypothesis. A two-tailed test of hypothesis will reject the null hypothesis if the sample statistic is significantly higher or lower than the population parameter. In a two-tailed test of hypothesis the rejection region is located on both the tails and the size of the rejection region is .025, whereas the central acceptance region is .95. In contrast to the two-tailed hypothesis, you would notice that in a one-tailed hypothesis the rejection region will be located only on one tail. Similarly, you would need to carry out the Type I and Type II error tests. Designate Type I error as α (alpha), and Type II error as β (beta). As pointed out in the text above, you need to remember that both types of errors cannot be reduced simultaneously, as reduction in one leads to increase in the other, if the sample size remains unchanged. If you follow the text in Section 18.3, you would be able to carry out both sets of tests on the hypothesis of your research. Make sure to include them in your research work report.

18.4 Tests of Significance

i) Chi-square test (χ^2)

Chi-square[®] is probably the most commonly used of all non-parametric tests. It is applicable when data are nominal and grouped in categories. You can examine the difference between the observed and the expected frequencies.

$$\chi^2 = \sum ((O - E)^2 / E)$$

Where, O and E are the observed and expected frequencies respectively.

The calculated value of χ^2 is compared with the table value of χ^2 for given degrees of freedom at a certain specified level of significance (e.g. 5%). If the calculated value of χ^2 is higher than the table value of χ^2 , then the difference between the theory and observation is considered to be significant. On the other hand, if the calculated value of χ^2 is lower than the table value of χ^2 , then the difference between the theory and observation is considered to be non-significant.

As mentioned above, while comparing the calculated value of χ^2 with the table value of χ^2 , one has to determine the degrees of freedom. Degree of freedom is the number of classes to which the values can be allocated at will or arbitrarily without defying the limitations or restrictions. For instance, if one has to choose four numbers whose sum is 100, the freedom of choice exists only for selecting three numbers, and the fourth is selected automatically. If, for example, the first three numbers are 14, 26, 32, then the fourth is fixed and must be 28 (100 - (14+ 26+ 32)). In this case the degree of freedom is three. Chi-square is used for a variety of purposes. Also there are numerous tests that are close to χ^2 . Here, the test of goodness of fit and the test of homogeneity / association are presented.

ii) **Test of goodness of fit:** We often want to know whether the observed frequencies are in agreement with the probability or expected theoretical distribution or not. The following steps may be followed:

Step 1: Define null and alternative hypotheses.

Step 2: Decide probability level.

Step 3: Estimate the expected frequency E for each category based on theory and or probability.

Step 4: Calculate chi-square.

Step 5: Determine the degree of freedom.

Step 6: Compare the observed chi-square with the tabulated chi-square. Accept/ reject the null hypothesis.

See Box 18.1 for an example.

Box 18.1 Example: Test whether a form of transport is favoured more significantly than another?

Frequencies	Mode of Transport					Total
	Car	Bus	Metro	Scooter	Train	
Observed	18	21	19	20	22	100
Expected*	20	20	20	20	20	100

Solution:

Step 1: Null hypothesis: There is no significant difference in the choice of the type of transportation.

Alternative hypothesis: There is significant difference in choice of type of transportation.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: The expected frequencies (20) in all the categories is based on the fact that there is an equal choice of the type of transportation.

Step 4: Calculations:

$$\chi^2 = \sum((O - E)^2 / E)$$

$$\chi^2 = ((18 - 20)^2 / 20) + ((21 - 20)^2 / 20) + ((19 - 20)^2 / 20) + ((20 - 20)^2 / 20) + ((22 - 20)^2 / 20)$$

$$\chi^2 = 4/20 + 1/20 + 1/20 + 0 + 4/20 = 10/20 = 0.5$$

Step 5: Degree of freedom = k - 1 = 5 - 1 = 4

Step 6: The table value of chi-square at 5% probability level for 4 degree of freedom is = 9.49. The calculated value of χ^2 (0.5) is lower than the table value of χ^2 (9.49). Thus the null hypothesis is accepted and the difference between the theory and observation is non-significant and there is no significant difference in the choice of the type of transportation.

iii) **Test of association/ homogeneity:** This type of χ^2 is used for two purposes. The first purpose is to examine whether or not the two or more attributes are associated (test of association). The second purpose is to determine whether two samples are drawn from the same population or not (test of homogeneity[®]). In the former case the data is based on

one sample whereas in the latter, there are two or more samples.

Chi-square, a non-parametric test, is a rough estimate of confidence; it accepts weaker, less accurate data as input than the parametric tests, like t-tests and the analysis of variance, and therefore, has less status in the pantheon of statistical tests. Nonetheless, its limitations are also its strengths; because chi-square is more 'forbearing' in the data it will accept, it can be used in a wide variety of researches.

The steps in the chi-square method for the test of homogeneity remain the same as that of the test of goodness of fit, except that in step 3 the expected frequencies are calculated for each cell as illustrated.

Populations	Attribute			Total
	Category 1	Category 2	Category 3	
Population 1	A	B	C	N_1
Population 2	D	E	F	N_2
Total	N_3	N_4	N_5	N

Expected Frequency of Cell A = $(N_1 * N_3) / N$

Expected Frequency of Cell B = $(N_1 * N_4) / N$

Expected Frequency of Cell C = $(N_1 * N_5) / N$

Expected Frequency of Cell D = $(N_2 * N_3) / N$

Expected Frequency of Cell E = $(N_2 * N_4) / N$

Expected Frequency of Cell F = $(N_2 * N_5) / N$

See Box 18.2 for an example to find out if there was a difference in the income of the two groups. On the basis of this example, you may take up another case to test association. Homogeneity.

Populations	Income groups		
	High	Middle	Low
Bhils	28	41	65
Minas	31	43	55

Solution:

Step 1: Null hypothesis: There is no significant difference in income between Bhils and Minas.

Alternative hypothesis: There is significant difference in income between Bhils and Minas.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: The expected frequencies are as given below.

Populations	Income groups						Total
	High		Middle		Low		
	Observed	Expected	Observed	Expected	Observed	Expected	
Bhils	28	30.06	41	42.80	65	61.14	134
Minas	31	28.94	43	41.20	55	58.86	129
Total	59	59.00	84	84.00	120	120.00	263

Expected Frequency of Cell A = $(N_1 * N_3) / N = 134 * 59 / 263 = 30.06$

Expected Frequency of Cell B = $(N_1 * N_4) / N = 134 * 84 / 263 = 42.80$

Expected Frequency of Cell C = $(N_1 * N_5) / N = 134 * 120 / 263 = 61.14$

Expected Frequency of Cell D = $(N_2 * N_3) / N = 129 * 59 / 263 = 28.94$

Expected Frequency of Cell E = $(N_2 * N_4) / N = 129 * 84 / 263 = 41.20$

Expected Frequency of Cell F = $(N_2 * N_5) / N = 129 * 120 / 263 = 58.86$

Step 4: Calculations:

$$\chi^2 = \sum ((O - E)^2 / E)$$

$$\chi^2 = ((28 - 30.06)^2 / 30.06) + ((41 - 42.80)^2 / 42.80) + ((65 - 61.14)^2 / 61.14) + ((31 - 28.94)^2 / 28.94) + ((43 - 41.20)^2 / 41.20) + ((55 - 58.86)^2 / 58.86)$$

$$= 0.141 + 0.076 + 0.244 + 0.147 + 0.079 + 0.253 = 0.940$$

Step 5: Degree of freedom = [(No. of rows - 1) * (No. of column - 1)] = (2-1)*(3-1) = 2

Step 6: The table value of chi-square at 5% probability level for 2 degree of freedom is = 5.991. The calculated value of χ^2 (0.940) is lower than the table value of χ^2 (5.991). Thus the null hypothesis is accepted and the difference between the theory and observation is non-significant and there is no significant difference in the income of Bhils and Minas.

There is a short cut method for the calculation of χ^2 if the frequency distribution is arranged in '2x2 contingency table', as illustrated in Figure 18.3.

	Variable 1 Category 1	Variable 2 Category 2	Total
Sample 1	A	B	A + B
Sample 2	C	D	C + D
Total	A + C	B + D	N = A + B + C + D

Figure 18.3 Short-cut Method to Calculate

$$\chi^2 = N * (A * D - B * C)^2 / (A + B) * (C + D) * (A + C) * (B + D)$$

The calculated value of χ^2 is examined against the tabulated value at 1 d. f. at specified probability level to ascertain significance. See Box 18.3 to find out significant difference between males and females in terms of their occupations.

Box 18.3 Example to Examine Significant Gender-based Difference in Occupation as Skilled/ Unskilled Labourers

Gender	Skilled Labourers	Unskilled Labourers
Males	47	56
Females	32	71

Solution

Step 1: Null hypothesis: There is no significant sex difference in skilled and unskilled laborers.

Alternative hypothesis: There is a significant sex difference in skilled and unskilled laborers.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: Calculations:

Gender	Skilled Labourers	Unskilled Labourers	Total
Males	47	56	103
Females	32	71	103
Total	79	127	206

$$N=206 \quad A*D= 3337 \quad B*C = 1792$$

$$A + B= 103 \quad C + D=103 \quad A + C=79 \quad B + D=127$$

$$\chi^2 = N * (A*D - B*C)^2 / (A + B) * (C + D) * (A + C) * (B + D)$$

$$\chi^2 = 206 * (3337 - 1792)^2 / 103 + 103 + 79 + 127$$

$$\chi^2 = 491727150 / 106440097 = 4.620$$

Step 4: Degree of freedom = [(No. of rows -1)* (No. of column -1)] = (2-1)*(2-1) = 1

Step 5: The table value of chi-square at 5% probability level for 1 degree of freedom is = 3.841. The calculated value of χ^2 (4.620) is higher than the table value of χ^2 (3.841).

So you can say as a conclusion that the null hypothesis is rejected and the sex difference between skilled and unskilled laborers is significant.

iv) Student's t test (t)

Student's t test is a parametric test most suitable for a small sample. It is probably the most widely used statistical test and certainly the most widely known. It is simple, straightforward, easy to use, and adaptable to a broad range of situations. No statistical toolbox should ever be without it. "Student" (real name: W. S. Gossett) developed the statistical methods to solve problems stemming from his employment in a brewery. Like chi-square, the following steps may be followed for the use of the Student's t test:

Step 1: Define null and alternative hypotheses.

Step 2: Decide probability level.

Step 3: Calculate the value of t using appropriate formula.

Step 4: Determine the degree of freedom.

Step 5: Compare the observed chi-square with the tabulated chi-square. Accept or reject the null hypothesis.

Student's t test is applied in different conditions, such as

- a) To test the significance of the mean of a random sample
- b) To test the difference between the means of the two independent samples
- c) To test the difference between the means of the two dependent samples
- d) To test the significance of the correlation coefficient.

Let us discuss each of the above conditions.

a) To test the significance of the mean of a random sample: This test is used when the researcher is interested in examining whether the mean of a sample from the normal population deviates significantly from the hypothetical population mean. The following formula is used for its calculation:

$$t = \{(M - \mu) * \sqrt{n}\} / S$$

When using actual mean:

$$S = \sqrt{[\sum(X - M)^2 / (n - 1)]}$$

When using assumed mean

$$S = \sqrt{[\sum d^2 - (d_m)^2 * n] / (n - 1)}$$

Where M and μ are the means of the sample and population respectively; n is the sample size

S is the standard deviation of the sample.

d = X - A, X being the variable

d_m is the mean of deviation

A is the assumed mean. Let us take an example, in Box 18. 4, of testing the mean nutritional intake.

Box 18.4 Example to Test The Mean Nutritional Intake in the Population with 2000 Calories.

Nutritional Intake (Calories)									
2300	2000	2150	1950	2000	2150	1900	1900	2250	2050

Solution:

Step 1: Null hypothesis: The mean nutritional intake in the population, from which the sample is drawn, is 2000 Calories.

Alternative hypothesis: The mean nutritional intake in the population, from which the sample is drawn, is not 2000 Calories.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: Calculations:

Nutritional Intake (Calories)	d = X - A	d ²
2300	300	90000
2000	0	0
2150	150	22500
1950	-50	2500
2000	0	0
2150	150	22500
1900	-100	10000
1900	-100	10000
2250	250	62500
2050	50	2500
20650	650	222500

$$n = 10 \quad \Sigma d^2 = 222500 \quad d_m = 650 / 10 = 65$$

$$M = 20650 / 10 = 2065 \quad \mu = 2000$$

$$S = \sqrt{[\{\Sigma d^2 - (d_m)^2 \cdot n\} / (n - 1)]}$$

$$S = \sqrt{[\{222500 - (65)^2 \cdot 10\} / 9] = 141.52}$$

$$t = \{(M - \mu) \cdot \sqrt{n}\} / S = \{(2065 - 2000) \cdot \sqrt{10}\} / 141.52 = 1.452$$

Step 4: Degree of freedom = 10 - 1 = 9

Step 5: The table value of t at 5% probability level for 9 degree of freedom is = 2.232. The calculated value of t (1.452) is lower than the table value of t (2.232).

Thus the null hypothesis is accepted and the difference is not significant.

b) To test the difference between the means of the two independent samples: This test is used when the researcher is interested in examining whether the respective means of two independent samples differ significantly from each other. The following formula is used for its calculation:

$$t = \{(M_1 - M_2) \cdot \sqrt{\{(n_1 \cdot n_2) / (n_1 + n_2)\}}\} / S$$

When using actual mean:

$$S = \sqrt{[\{\Sigma(X_1 - M_1)^2 + \Sigma(X_2 - M_2)^2\} / (n_1 + n_2 - 2)]}$$

When using assumed mean

$$S = \sqrt{[\{\Sigma d_1^2 + \Sigma d_2^2 - n_1 (M_1 - A)^2 - n_2 (M_2 - A)^2\} / (n_1 + n_2 - 2)]}$$

Where $d_1 = X_1 - A_1$ and $d_2 = X_2 - A_2$ respectively

M_1 and M_2 are the respective means of the two samples

A_1 and A_2 are the assumed mean of the two samples

n_1 and n_2 are the sample sizes and S is the common standard deviation. We would take an example in Box 18.5 to find out the marital distance among the Santhals and Murias.

Box 18.5 Example to Examine whether Santhals and Murias Differ in Marital Distance										
	Marital Distance (km)									
Santhals	10	12	15	17	18	17	19	22	22	12
Murias	22	19	21	23	18	21	23	20	19	21

Solution:

Step 1: Null hypothesis: Santhals and Murias do not differ in their marital distance.

Alternative hypothesis: Santhals and Murias differ in their marital distance.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: Calculations:

Santhals			Murias			$A_1 =$	$A_2 =$
X_1	$d_1 = X_1 - A_1$	d_1^2	X_2	$d_2 = X_2 - A_2$	D_2^2		
10	-6	36	22	2	4	16	20
12	-4	16	19	-1	1	16	20
15	-1	1	21	1	1	16	20
17	1	1	23	3	9	16	20
18	2	4	18	-2	4	16	20
17	1	1	21	1	1	16	20
19	3	9	23	3	9	16	20
22	6	36	20	0	0	16	20
22	6	36	19	-1	1	16	20
12	-4	16	21	1	1	16	20
164	4	156	207	7	31		

$$A_1 = 16 \quad A_2 = 20 \quad M_1 = 16.4 \quad M_2 = 20.7$$

$$n_1 = 10 \quad n_2 = 10 \quad \sum d_1^2 = 156 \quad \sum d_2^2 = 31$$

$$S = \sqrt{[\{\sum d_1^2 + \sum d_2^2 - n_1 (M_1 - A_1)^2 - n_2 (M_2 - A_2)^2\} / (n_1 + n_2 - 2)]}$$

$$S = \sqrt{[\{156 + 31 - 10 (16.4 - 16)^2 - 10 (20.7 - 20)^2\} / (10 + 10 - 2)]}$$

$$S = \sqrt{[\{156 + 31 - 10 (16.4 - 16)^2 - 10 (20.7 - 20)^2\} / (10 + 10 - 2)]}$$

$$S = \sqrt{[180.5 / 18]} = \sqrt{10.028} = 3.167$$

$$t = [(M_1 - M_2) * \sqrt{(n_1 * n_2) / (n_1 + n_2)}] / S$$

$$t = \{(16.4 - 20.7) * \sqrt{(100 / 20)}\} / 3.167 = (4.3 * 2.236) / 3.167 = 3.036$$

Step 4: Degree of freedom = 10 + 10 - 2 = 18

Step 5: The table value of t at 5% probability level for 9 degree of freedom is = 2.101. The calculated value of t (3.036) is higher than the table value of t (2.101).

You can say that the null hypothesis is rejected and the difference in marital distance between Santhals and Murias is significant.

c) **To test the difference between the means of the two dependent samples:** This test is used when the researcher is interested in examining whether the mean of two dependent samples differ significantly from each other. The following formula is used for its calculation:

$$t = (d_m * \sqrt{n}) / S$$

$$S = \sqrt{[\sum (d - d_m)^2 / (n - 1)]}$$
 or

$$S = \sqrt{[(\sum d^2 - (d_m)^2 * n) / (n - 1)]}$$

Where, $d = X_1 - X_2$;

d_m is the mean of the deviations;

n_1 and n_2 are the sample sizes; and

S is the common standard deviation. We would take an example in Box 18.6 to find out differences in observations of two researchers.

Box 18.6 Example: Two observers have taken the income of ten households. Examine whether their observations differ significantly?

	Household No.									
Observers	1	2	3	4	5	6	7	8	9	10
Observer 1	2400	1950	2200	1800	2050	2250	2000	1950	2300	2000
Observer 2	2300	2000	2150	1950	2000	2150	1900	1900	2250	2050

Solution:

Step 1: Null hypothesis: The difference in the observation by the two observers is not significant.

Alternative hypothesis: The difference in the observation by the two observers is significant.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: Calculations:

Household No.	Observer 1	Observer 2	$d = X_1 - X_2$	d^2
1	2400	2300	100	10000
2	1950	2000	-50	2500
3	2200	2150	50	2500
4	1800	1950	-150	22500
5	2050	2000	50	2500
6	2250	2150	100	10000
7	2000	1900	100	10000
8	1950	1900	50	2500
9	2300	2250	50	2500
10	2000	2050	-50	2500
			250	67500

$$n = 10 \quad d_m = 250/10 = 25$$

$$S = \sqrt{[\sum d^2 - (d_m)^2 * n / (n - 1)]}$$

$$S = \sqrt{[(67500 - (25)^2 * 10) / 9]}$$

$$S = 82.496$$

$$t = (d_m * \sqrt{n}) / S$$

$$t = (25 * \sqrt{10}) / 82.496 = 0.958$$

Step 4: Degree of freedom = 10 - 1 = 9

Step 5: The table value of t at 5% probability level for 9 degree of freedom is = 2.232. The calculated value of t (0.958) is lower than the table value of t (2.232).

You would say that the null hypothesis is accepted and the difference between the observers is not significant.

d) To test the significance of Correlation coefficient: Whether a coefficient of correlation is significant or not may be tested using the following formula:

$$t = \frac{r * \sqrt{(n - 2)}}{\sqrt{(1 - r^2)}}$$

Where, r is the coefficient of correlation and n is the number of observations. Degree of freedom is n-2. In Box 18.7, we would take an example to test the importance of a correlation.

Box 18.7 Example: Using the Following Data to Test the Significance of the Correlation

$$r = 0.45, n = 102$$

Step 1: Null hypothesis: The coefficient of correlation is not significant.

Alternative hypothesis: The coefficient of correlation is significant.

Step 2: Probability level for the hypothesis testing is 5%.

Step 3: Calculations:

$$t = (r * \sqrt{(n - 2)}) / \sqrt{(1 - r^2)}$$

$$t = (0.45 * \sqrt{(100)}) / \sqrt{(1 - 0.45^2)}$$

$$t = (0.45 * 10) / \sqrt{(1 - 0.2025)} = 4.5 / \sqrt{0.7975} = 4.5 / 0.893 = 5.039$$

Step 4: Degree of freedom = 102 - 2 = 100

Step 5: The table value of t at 5% probability level for 100 degree of freedom is = 1.96. The calculated value of t (5.039) is higher than the table value of t (1.96).

Thus, you would find that the null hypothesis is rejected and the correlation is significant.

Reflection and Action 18.3

Of the four tests in section 18.4, select one test and carry it out with respect to your own research work. Write it out in detail in your research work report.

18.5 Conclusion

Unit 18 has provided you with a range of ways to draw inferences. There are a good number of examples given for you to try and prepare your own examples. The exercises of working with as many as possible examples would help you to master the skills of testing hypotheses and estimating unknown parameters of the population. You need to keep in mind that no matter what design you used to test a hypothesis, you would reach only approximations in terms of probability. The testing of a hypothesis prepares for you the ground for generating further hypotheses and in this manner the scientific knowledge progress. Initial approximations put on firm basis the original hypothesis and from this you can further deduce other hypotheses. If you are able to establish links between propositions you would have generated scientific knowledge.

Further Reading

Handel, J.D. 1978, *Statistics for Sociology*, Englewood Cliffs, N.J.

Watson, G. and McGawd 1980. *Statistical Inquiry: Elementary Statistics for the Political Science and Policy Sciences*. John Wiley: New York

